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**ELECTRONICS  
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**ELECTRONICS ENGINEERING**  
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## CHAPTER-1

### BINARY SYSTEM

#### Base Conversion

A number  $a_n, a_{n-1} \dots a_2, a_1, a_0, a_{-1}, a_{-2}, a_{-3} \dots$  expressed in a base  $r$  system has coefficient multiplied by powers of  $r$ .

$$a_n r^n + a_{n-1} r^{n-1} + a_{n-2} r^{n-2} + \dots + a_1 r + a_0 + a_{-1} r^{-1} + a_{-2} r^{-2} + a_{-3} r^{-3} + \dots \quad \dots(A)$$

#### Key Points:

To convert a number of base  $r$  to decimal is done by expanding the number in a power series as in (A)  
Then add all the terms.

**Example :** Convert following Binary number  $(11010.11)_2$  in to decimal number.

**Solution:** Base  $r = 2$

$$1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$$

$$(11010.11)_2 = (26.75)_{10}$$

**Example :** Convert  $(127.4)_8$  in to decimal equivalent.

$$\text{Solution: } 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = (87.5)_{10}$$

#### Numbers with Different bases:

Decimal (r = 10)	Binary (r = 2)	Octal (r = 8)	Hexadecimal (r = 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

**Example:** Convert following hexadecimal number into decimal number:  $(B65F)_{16}$

**Solution:**

$$11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0 = (46687)_{10}$$

**Conversion of decimal number to a number in base r:**

- Separate the number into an integer part and fraction part.
- Divide the number and all successive quotients by r and accumulating the remainders.
- Conversion of decimal fraction is done by multiplying the fraction and all successive fractions by r and integers are accumulated.

**Example:** Convert decimal number 41 to binary.

**Solution:**

	Integer quotient		Remainder	Coefficient
41/2 =	20	+	1	$a_0 = 1$
20/2 =	10	+	0	$a_1 = 0$
10/2 =	5	+	0	$a_2 = 0$
5/2 =	2	+	1	$a_3 = 1$
2/2 =	1	+	0	$a_4 = 0$
1/2 =	0	+	1	$a_5 = 1$

$$(41)_{10} \rightarrow (101001)_2$$

**Example:** Convert  $(153)_{10}$  to octal.

**Solution:**

	Integer quotient		Remainder	Coefficient
153/8 =	19	+	1	$a_0 = 1$
19/8 =	2	+	3	$a_1 = 3$
2/8 =	0	+	2	$a_2 = 2$

Thus  $(153)_{10} \rightarrow (231)_8$

**Example:** Convert  $(0.6875)_{10}$  to Binary.

**Solution:** 0.6875 is multiplied by 2 to give an integer and a fraction. The new fraction is multiplied by 2 to give a new integer and new fraction.

This process is continuing until the fraction becomes zero or until the numbers of digits have sufficient accuracy.

	Integer		Fraction		Coefficient
$0.6875 \times 2$	=	1	+	0.3750	$a_{-1} = 1$
$0.3750 \times 2$	=	0	+	0.7500	$a_{-2} = 0$
$0.7500 \times 2$	=	1	+	0.5000	$a_{-3} = 1$
$0.500 \times 2$	=	1	+	0.0000	$a_{-4} = 1$

$$(0.6875)_2 \rightarrow (0.1011)_2$$

**Example:** Convert  $(0.513)_{10}$  to octal.

**Solution:**

$0.513 \times 8$	=	4	+	0.104	$a_{-1} = 4$
$0.104 \times 8$	=	0	+	0.832	$a_{-2} = 0$
$0.832 \times 8$	=	6	+	0.656	$a_{-3} = 6$
$0.656 \times 8$	=	5	+	0.248	$a_{-4} = 5$
$0.248 \times 8$	=	1	+	0.984	$a_{-5} = 1$
$0.984 \times 8$	=	7	+	0.872	$a_{-6} = 7$

Answer to six significant figures is:

$$(0.406517\dots)_8$$

Thus  $(0.513)_{10} \rightarrow (0.406517)_8$

$$(41.6875)_{10} \rightarrow (101001.1011)_2$$

$$(153.513)_{10} \rightarrow (231.406517)_8$$

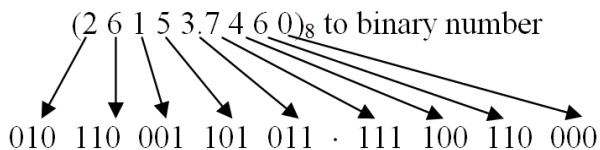
**Octal and hexadecimal numbers:**

Conversion from binary to octal is easily done by partitioning the binary number into groups of 3 digits each starting from binary point & proceeding to left and to the right.

The corresponding octal digit is then assigned to each group.

For conversion into hexadecimal, binary number is divided into group of 4 digits.

**Example :**



Thus binary number is

$$(10 110 001 101 011.11110011)_2$$

**Example:** Convert binary to hexadecimal number:

$$(10\ 1100\ 0110\ 1011.1111\ 0010)_2$$

$$0010\ 1100\ 0110\ 1011.1111\ 0010$$

$$2\ C\ 6\ B\ F\ 2 = (2C6B.F2)_{16}$$

**Example :**  $(673.124)_8$  to binary number:

$$(673.124)_8 \equiv (110\ 111\ 011 \cdot 001\ 010\ 100)_2$$

$$6\ 7\ 3\ 1\ 2\ 4$$

$(306.D)_{16}$  to binary number:

$$(306.D)_{16} \equiv (0011\ 0000\ 0110.1101)_2$$

$$3\ 0\ 6\ D$$

### Complements

Complements are used in digital computer for simplifying the subtraction operations and for logic manipulation. There are 2 types of complements for each base  $r$  system

i. Diminished radix complement  $((r - 1)$ 's complement

ii. Radix complements  $(r)$ 's complement

i. Diminished radix complement:

- Given a number  $N$  in base  $r$  having  $n$  digits, the  $(r - 1)$ 's complement of  $N$  is defined as  $(r^n - 1) - N$ .
- For decimal number  $r = 10$ ,  $(r - 1)$ 's complement or 9's complement of  $N$  is  $(10^n - 1) - N$ .

**9's complement:  $(10^n - 1) - N$  :**

- $10^n$  can be represented as single 1 followed by  $n$  0's
- $10^n - 1$  is number represented by  $n$  9's.
- Thus 9's complement can be obtained by subtracting each digit of number  $N$  by  $n$  9's.

**Example :** Find 9's complement of 546700

**Solution:**

$$999999 - 546700 = 453299$$

9's complement of 546700 is 453299

**1's Complement for binary number:**

- It is given as  $(2^n - 1) - N$
- $2^n$  can be representing as binary number consist of single 1 followed by  $n$  0's.
- $2^n - 1$  can be represented as  $n$  1's.

**Example :** 1's complement of 1011000.

**Solution:**  $1111111 - 1011000 = 0100111$

**Note:** It is similar to changing 1's to 0's and 0's to 1 or complement each digit of number is similar to taking 1's complement of the number.

**Note:**  $(r - 1)$ 's complement of octal and hexadecimal number is obtained by subtracting each digit from 7 and F respectively.

### (ii) Radix Complement:

$r$ 's complement of  $n$  digit number  $N$  in base  $r$  is defined as  $r^n - N$  for  $N \neq 0$  & 0 for  $N = 0$

It is equivalent to adding 1 to  $(r - 1)$ 's complement.

If  $(r - 1)$ 's complement is given,  $r$ 's complement can be obtained by adding 1 to  $(r - 1)$ 's complement.

**Example :** Find 10's complement of number if its 9's complement is 453299.

**Solution:**  $r$ 's complement is  $453299 + 1$

$$r\text{'s complement} = 453300$$

**Example :** 2's complement of 1010110 is:

**Solution:** 1's complement: complement each digit of number  $(1010110) \rightarrow (0101001)_2$

Thus 2's complement is  $0101001 + 1$

$$2\text{'s complement} = (0101010)_2$$

### Another Method to Obtain 10's, 2's Complement:

Leaving all least significant 0's unchanged, subtracting the first non-zero least significant digit from 10 and subtracting all higher significant digits from 9.

**Example:** Find 10's complement of 012398.

**Solution:**

- Subtract 8 from 10 in the least significant position
- Subtracting all other digits from 9.

$$\begin{array}{r} 9999910 \\ - 012398 \\ \hline 987602 \end{array}$$

Thus 10's complement of 012398 is 987602.



**Example:** 10's complement of 246700.

**Solution:** Leaving 2 least significant 0's unchanged, subtracting 7 from 10 and other 3 digits from 9.

$$\begin{array}{r} 9991000 \\ -246700 \\ \hline 753300 \end{array}$$

Thus 10's complement of 246700 is 753300

**Similarly 2's complement** can be formed by leaving all least significant 0's and first 1 unchanged and replacing 1's with 0's and 0's with 1's in all other higher significant digits.

**Example:** 2's complement of  $(1101100)_2$ :

**Solution:**

$$\begin{array}{r} 1101\ 1\ 00 \\ \swarrow \quad \searrow \quad \searrow \\ \text{Remain unchanged} \\ \text{Remain unchanged} \\ \text{Reverse all digits} \\ 0010100 \end{array}$$

Thus 2's complement of 1101100 is  $(0010100)_2$

### Subtraction with complement:

- i. Convert subtrahend N to r's complement.
- ii. Then add to the minuend M.
- iii. If  $M \geq N$ , sum will produce end carry, which can be discarded, what is left is the result,  $M - N$ .
- iv. If  $M < N$ , sum does not produce carry and is equal to  $r^n - (N - M)$ , which is same as r's complement of  $(N - M)$ .
- v. To take the answer in familiar form, take the r's complement of the sum and place a negative sign in front.

**Example:** Using 10's complement, subtract  $72532 - 3250$

**Solution:**

$$\begin{array}{l} M = 72532 \\ N = 03250 \end{array}$$

10's complement of N = 96750

$$\begin{array}{r} \text{Sum: } 72532 \\ + 96750 \\ \hline 169282 \end{array}$$

Discard end carry as  $M > N$  so result: 69282

**Example:** Using 10's complement, subtract  $3250 - 72532$

**Solution:**  $M = 3250$

$$N = 72532$$

10's complement of 72532 is

$$\begin{array}{r} 9999 \ 10 \\ - 7253 \ 2 \\ \hline \end{array}$$

10's complement 27468

Sum:  $3250$

$$\begin{array}{r} 27468 \\ \hline \end{array}$$

Sum  $30718$

Since  $N > M$  so no end carry.

Therefore answer is  $-(10's \text{ complement of } 30718) = -69282$

**Example:** Subtract  $1010100 - 1000011$

**Solution:** 2's complement of N (1000011)=0111101

Sum:  $1010100$

$$\begin{array}{r} + 0111101 \\ \hline \end{array}$$

$10010001$

So result is 0010001

Note: Subtraction can also be done using  $(r - 1)$ 's complement.

### Signed Binary numbers

When binary number is signed, left most bit represents the sign and rest of bits represent the number.

- If binary number is unsigned, then left most bits is the most significant bit of the number.
- Positive or Negative can be represented by (0 or 1) bit which indicate the sign.

**Example:** String of bits 01001 can be considered as 9 (unsigned binary) or +9 (signed binary) because left most bits are 0.

**Example:** String of bits 11001 represent 25 when considered as unsigned number or  $-9$  when considered as signed number.

### Negative number representation:

(i) **Signed magnitude representation:** In this representation number consist of a magnitude and a symbol (+ or -) or bit (0 or 1) indicating the sign, left most bit represents sign of a number.

$$11001 \rightarrow -9$$

$$01001 \rightarrow +9$$

(ii) **Signed complement system:**

- In this system, negative number is indicated by its complement.

- It can use either 1's or 2's complement, but 2's complement is most common.

**Note:**

- 2's complement of positive number remain number itself.
- In both signed magnitude & signed complement representation, the left most significant bit of negative numbers is always 1.

**Example :** +9 00001001  
 -9 11110111 (2's complement of +9)

**Note:** Signed complement of number can be obtained by taking 2's complement of positive number including the sign bit.

- Signed magnitude system is used in ordinary arithmetic, cannot employed in computer arithmetic because of separate handling of the sign and the magnitude.
- In computer arithmetic signed complement system is used to represent negative numbers.

Decimal	Signed 2'Complement	Signed 1's complement	Signed magnitude
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	-	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100

**Note:**

2's complement representation range for n bit number is:

$$-2^{n-1} \text{ To } 2^{n-1} - 1 \quad (\text{for } n = 8 \text{ range is } + 127 \rightarrow - 128)$$

$$1's \text{ complement: } -(2^{n-1} - 1) \text{ to } (2^{n-1} - 1)$$

$$\text{Signed magnitude range: } -(2^{n-1} - 1) \text{ to } (2^{n-1} - 1)$$

**Binary Codes**

Any discrete element of information distinct among a group of quantities can be represented with a binary code.

- n bit binary code is a group of n bits that have  $2^n$  distinct combinations of 1's and 0's with each combination representing one element of the set that is being coded.

**Example:** With 2 bits  $2^2 = 4$  elements can be coded as: 00, 01, 10, 11

With 3 bits  $2^3 = 8$  elements can be coded as:

000, 001, 010, 011, 100, 101, 110, 111

- Minimum number of bits required to code  $2^n$  distinct quantities in n.
- The bit combination of an n bit code is determined from the count in binary from 0 to  $2^n - 1$ .

**Example :** 3 bit combination

000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

**BCD (Binary coded decimal)**

- A number with k decimal digits require 4 K bits in BCD.
- A decimal number in BCD is same as its equivalent binary number only when number is between 0 to 9.
- BCD number needs more bits than its equivalent binary.
- Example:  $(185)_{10} = (000110000101)_{BCD} = (1011001)_2$
- In BCD number, each bit is represented by its equivalent binary representation.

**Note:** BCD numbers are decimal numbers and not binary numbers, although they use bits in their representation.

- Decimal are written as 0,1,2,3,...,9 which BCD can be written as : 0000, 0001, 0010, 0011, ..., 1001

**Benefits of BCD:**

- BCD helps to do arithmetic operation directly on decimal numbers without converting them into equivalent binary numbers.

Decimal system	BCD digits	Binary equivalent
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	0111	0111
8	1000	1000
9	1001	1001
10	00010000	1010
11	00010001	1011

**BCD addition:**

- If binary sum is equal to or less than 1001 (without a carry), the corresponding BCD digit is correct.
- If binary sum  $\geq 1010$ , the result is an invalid BCD.
- Addition of  $6 = (0110)_2$  to the binary sum converts it to the correct digits and also produces a carry as required.

<b>Example:</b>	4	0100	4	0100	8	1000
	<u>+5</u>	<u>+0101</u>	<u>+8</u>	<u>+1000</u>	<u>+9</u>	<u>+1001</u>
	9	1001	12	1100	17	10001
				<u>+0110</u>		<u>0110</u>
				10010		10111

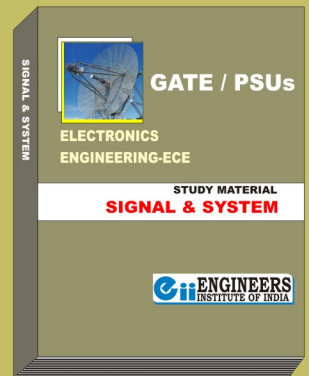
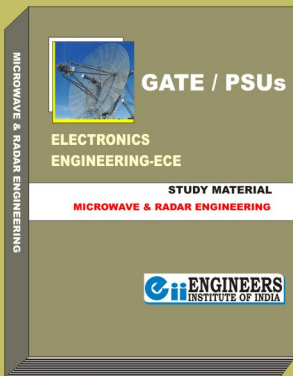
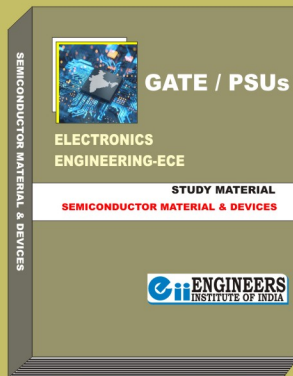
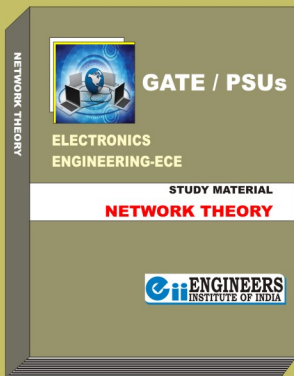
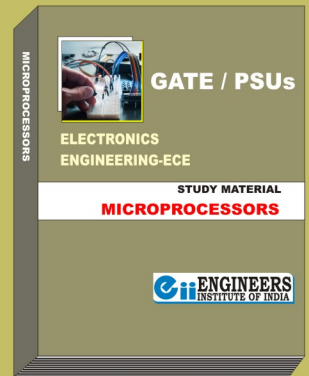
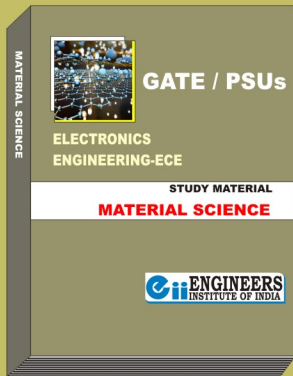
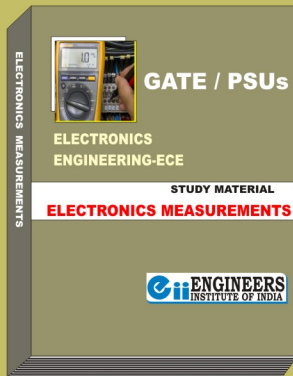
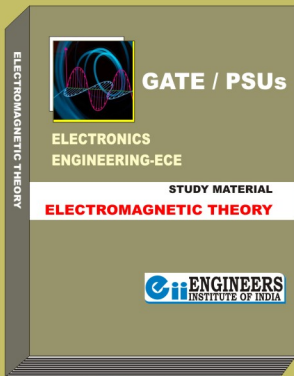
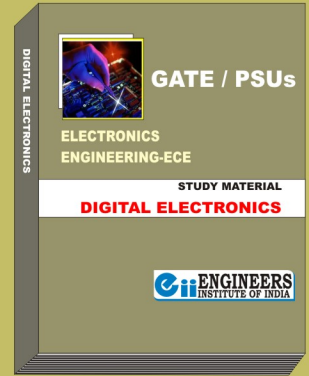
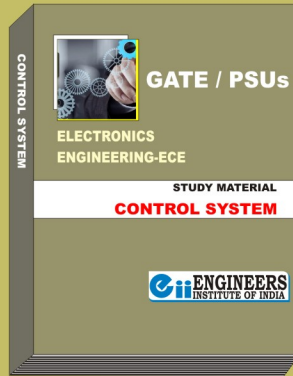
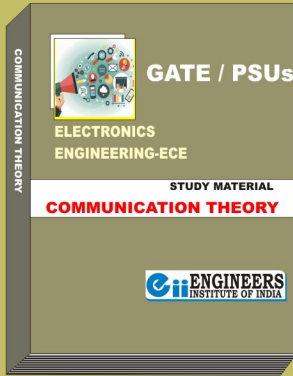
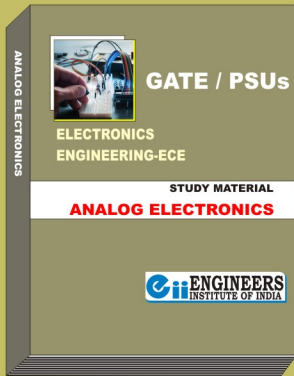
**Example:** Add  $184 + 576$  in BCD.

**Solution:**

	0001	1000	0100	184
	<u>0101</u>	<u>0111</u>	<u>0110</u>	<u>+576</u>
Binary sum	0110	1111	1010	
Add 6		<u>0110</u>	<u>0110</u>	
BCD sum	0111	0110	0000	
	7	6	0	760

- Representation of Signed decimal numbers in BCD is similar to the representation of signed number in binary.
- Sign of decimal number is represented with 4 bits :  
Positive number: '0000' (0)  
Negative number – '1001' (9)

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